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In this paper we investigate whether torsion can generate neutrino mass. We consider an Einstein-Dirac-Cartan lagrangian in 4 dimensional spacetime with torsion generated by the Kalb-Ramond antisymmetric field in presence of a bulk fermion in five dimensions.We show that such a coupling can generate a mass term for the neutrino.

I. INTRODUCTION

There have been numerous works [1–4] in recent times dealing with the generation of neutrino mass from higher dimensions. In particular, it has been argued that bulk fermions living in the higher dimensions could couple to fermionic fields living in 3+1 dimensions via the Higgs field and generate mass. There have been primarily two approaches: large scale compactification(hereafter referred to as ADD) [1] and Randall-Sundrum(RS)mechanism [5]. In this paper we shall deal primarily with the former approach in which,a large scale compactification has been shown to generate mass for neutrinos which is in agreement with the present experimental bound. However such a rough match is of course incapable of explaining the solar/atmospheric neutrino anomalies completely. We should search for other means to satisfactorily explain the anomaly.

A tempting place to look for an explanation is the geometry of space-time, in particular, torsion. In string theory, it is well known that the field strength corresponding to the second rank antisymmetric tensor field can be identified with spacetime torsion. It has been argued from gauge-theoretic view-point [6] that such an identification is necessary to preserve $U(1)$ gauge symmetries in space-time with torsion. In [7], it was shown that parity violating torsion could be an additional source for helicity flip. Moreover, work such as [8] show that there is a contribution to the oscillation-phase from torsion. This is sufficient to motivate the analysis of the contribution of torsion in the solar neutrino problem.

A plausible picture for a geometric contribution is hence via the antisymmetric H-field. In 3+1 dimensions this field can be related to the axion via duality. As such, this opens up a way to at least verify whether the known bounds for the axion are consistent with a neutrino-mass generation. We will consider a process via which a massless bulk-fermion, specifically a right-handed neutrino interacts via a localized H-field to produce a mass term for a left-handed neutrino living on the 3-brane. Normally, such gravitational interaction terms are Planck-suppressed. We will argue that for specific cases, these interaction terms can become crucial for neutrino-oscillation.

Gauge invariance dictate that the torsion field be thought of as being generated by the antisymmetric tensor field with Chern-Simons extension required for $U(1)$ gauge anomaly cancellation in string theory [6]. In [9] a scheme was given for coupling torsion to fermion fields via the Dirac-Einstein-Cartan (DEC)lagrangian with a more general parity-violating term. The effect of this parity violating term however will not be considered here. Thus if we write the connection as:

$$\bar{\Gamma}_{\alpha\beta\gamma} = \Gamma_{\alpha\beta\gamma} - \frac{1}{M} H_{\alpha\beta\gamma} \tag{1}$$

where M has the dimension of mass, we will get an interaction term[1] in the DEC lagrangian of the form:

$$\mathcal{L}_{\text{int}} = -\frac{i}{M}\bar{\psi}\gamma^{\mu}\sigma^{\nu\rho}H_{\mu\nu\rho}\psi \tag{2}$$

where γ's are the standard gamma-matrices and $\sigma^{\nu\rho} = i/4[\gamma^{\nu},\gamma^{\rho}]$. This is the term that will be considered as a source for generating masses for neutrinos.

We consider the antisymmetric field generating torsion to be localized on the brane. A massless bulk fermion, $\psi_R^B(x,y)$ living in five dimensions with one large extra dimension over the four dimensional spacetime. Here x denotes

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[1] Strictly speaking, there will be a Chern-Simon's term as well but since this is further Planck suppressed, we will drop it at this stage.

the coordinates x^μ for the four dimensional spacetime and y is the coordinate for the extra fifth dimension. This fermion is assumed to be a right-handed singlet and interacts with the neutrinos on the brane via the torsion field. The corresponding interaction term turns out to be:

$$\mathcal{L}_{\rm int} = -\frac{i}{M}\bar{\psi}(x)\gamma^\mu \sigma^{\nu\rho} H_{\mu\nu\rho}\psi_R^B(x, y = 0) \tag{3}$$

where the brane is localized at $y = 0$. Using the duality relation between the massless field $H_{\mu\nu\rho}$ and the axion χ,

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda}\partial^\lambda \chi \tag{4}$$

we get

$$\mathcal{L}_{\rm int} = -\frac{i}{M}\bar{\psi}(x)\,\gamma^\mu \sigma^{\nu\rho}\epsilon_{\mu\nu\rho\lambda}\partial^\lambda \chi\,\psi_R^B(x, y = 0) \tag{5}$$

where χ is the axion field. This simplifies to[2]:

$$\mathcal{L}_{\rm int} = \frac{2}{M}\bar{\psi}(x)\,\gamma_5\gamma_\lambda \partial^\lambda \chi\,\psi_R^B(x, y = 0) \tag{6}$$

Mode expanding the bulk fermion and retaining the lowest massive mode, we get on integrating by parts

$$\mathcal{L}_{\rm int} = -\frac{2m_B}{M}\bar{\psi}(x)\,\gamma_5 \chi\,\psi_R^B(x, y = 0) \tag{7}$$

where m_B is the mass of the bulk-fermion generated by compactification. Now we can replace the axion field by its vacuum expectation value. This is motivated by the Rohm-Witten quantization in [11,12] where it was shown that the antisymmetric tensor field H acquires a vev through the instanton corrections on the world sheet. Further using $\psi_L = 1/2(1 - \gamma_5)\psi$, where ψ belongs to a doublet, we get

$$\mathcal{L}_{\rm int} = \frac{2m_B}{M}\bar{\psi}_L(x)\,\chi\,\psi_R^B(x, y = 0) - \frac{2m_B}{M}\bar{\psi}_R(x)\,\chi\,\psi_R^B(x, y = 0) \tag{8}$$

This implies that the left-handed neutrino has obtained a mass

$$m_\nu = \frac{2m_B\langle\chi\rangle}{M} \tag{9}$$

Here M should be the 4-dimensional Planck constant. m_B should be related to the radius of compactification R by $m_B \sim 1/R$. Now if we consider an invisible axion [13,14] which was considered to be a good candidate for the cold dark matter component of the universe,$\langle\chi\rangle$ works out to be roughly 10^3 GeV. There are two phenomenologically viable cases that we consider:

- Using a large extra-dimension, where the Planck scale was effectively set to 1TeV(or R=1mm), we find $m_\nu \approx 10^{-20}$eV. This is a small correction to the mass generated by the Higg's field and will probably be undetectable.

- It has been shown recently by Dienes [16]that the masses of Kaluza-Klein modes depend on the shape-moduli. It was shown that for compactification on a sufficiently "stretched" 2-torus would allow the KK modes to become infinitely massive, which is perhaps phenomenologically desirable. The shape-moduli have no effect on the energy scales such as the effective Planck mass or the GUT scale since they depend only on the volume of the compact manifold. This inspires us to put $1/R \sim 1$TeV. This gives $m_\nu \approx 10^{-4}$eV which is roughly of the right order of magnitude for the neutrino masses.

It may be noted that although we do get a proper mass term for left-handed neutrino, we get a negative-mass term for the right-handed one. This is interesting since if other neutrino-mass generating mechanisms like the Higgs

[2]Such interaction terms were considered generically in [15]

coupling is considered, the torsion may be able to generate a mass term of comparable magnitude but with opposite sign, thereby creating an asymmetry. An explanation for this is not known at this point.

We conclude by summarizing our results. We have computed the geometric contribution to neutrino mass. For suitable shape-moduli, the contribution may even be comparable to the neutrino mass. Whereas our approach seems to yield interesting results, a more concrete approach could be to assume that H resides in the bulk. We intend to take this up in a future publication [17].

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